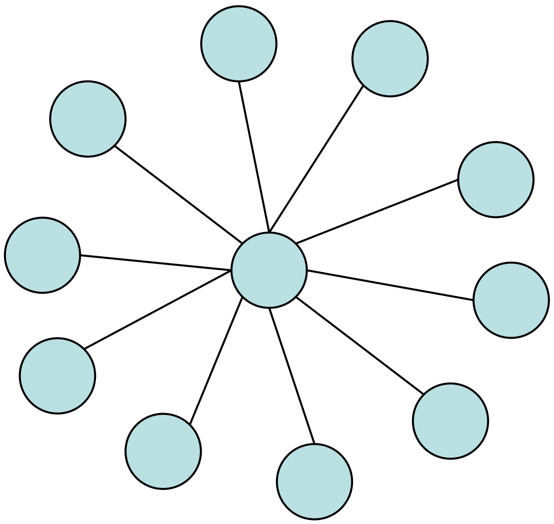
Strategic Network Formation



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Plenary Talk
GameNets 2009

Outline

1. Introduction: structure of networks
2. Strategic foundations
3. Unilateral linking

    Application: communication networks

4. Pairwise linking

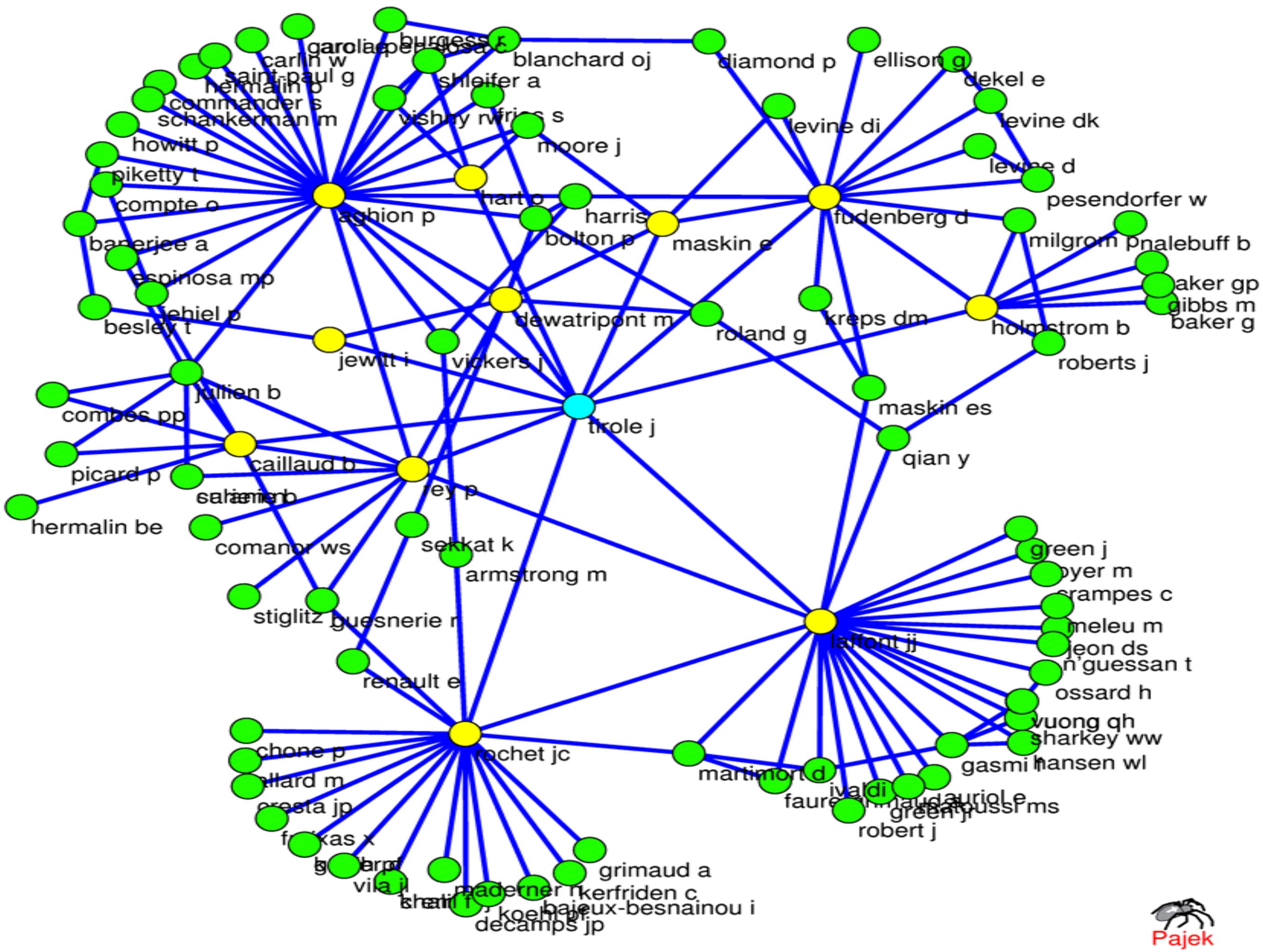
    Application: research alliances

5. Quick summary
6. Related themes
7. General theory: open problems

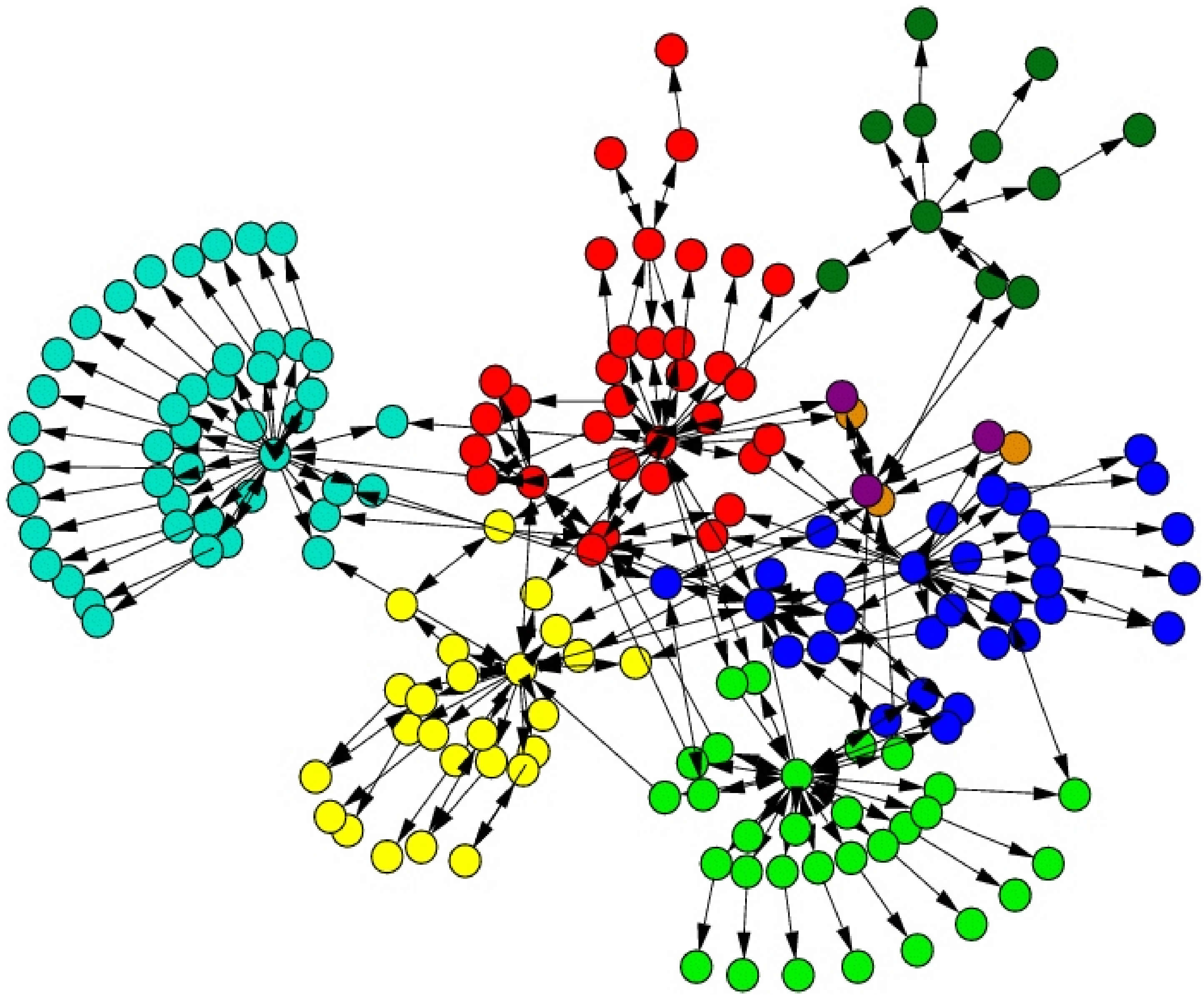
1. The structure of networks

- A network describes a collection of nodes and the links between them.
- Once you begin to study networks it is difficult not to see them everywhere.
- Examples: *Internet, World wide web, airline networks, friendships, cellular networks, research alliances, trade and defence alliances, co-authorships, trade & exchange, guanxi.*
- *What is the structure of these networks?*

Local network of J. Tirole in 1990's



Note: Some economists might appear twice or are missing due to the use of different initials or misspellings in EconLit.



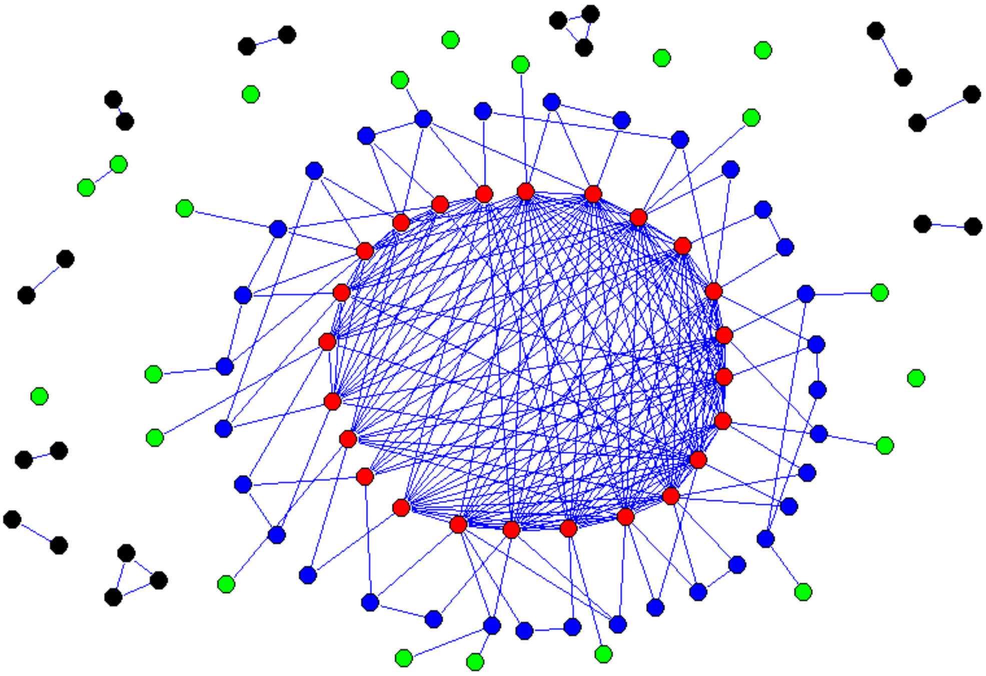


Figure 2.10: Research collaboration among firms

1. Structure of networks

- **Three key properties:**

- *Degree distribution*: *Average degree is very small & very unequal*. WWW: over 200 million web sites, av. degree 7.5, median <10, some pages have over hundred thousand links!

- *Clustering: Clustering is very high in social networks.* Friends of friends are also my friends… Economics co-author network the clustering coefficient 0.157; over 7,000 times of a random network.

- *Average distances: The average distance between nodes is very small.* WWW: 180 million web sites and the average distance is only 6. In firms alliance network over 4000 nodes, average distance is 4.

1. Structure of networks

- Social and economic networks display common features: *low average degree, very unequal degree distribution, clustering is high* and the *average distance* between nodes is *small*.
- **Small worlds**: Network with small av.degree, high clustering, & small av. distance is a *small world* by Watts and Strogatz (1998). Expression much older: e.g., Milgram (1967) experiment.

- **Key questions**: Who forms the networks? When do they have this structure? Why does it matter?

2. Strategic foundations of networks

Key features of linking:

1. **Linking is a deliberate decision**:
   Examples: Scientists decide on whether to collaborate
   Firms choose to form an alliance;
   I decide on hyperlink with your homepage.
2. **Externality/spillover**: Link between I and 2 affects payoffs of 3 as well as her rewards from new links.
   Examples: capacity constraints in co-authorship;
   firm A and B collaborate affects firm C.

Combine 1 & 2: Games of Network Formation.

2. Strategic foundations of networks

- Key issues in modelling:

1. Payoffs: linking generates rewards and entails costs. We define these formally.
2. Power: who decides on the link, one person, two persons, all players etc.
3. Information: what do I know -- about other players and about the network -- when I form a link?

We start with the simplest case: a player decides on whether to link with others. No transfers or bargaining. Full Information about rewards and costs of linking and about the network.

2. Strategic network games: antecedents

- Link formation & externalities: Boorman (1975).
- Communication networks in cooperative games, Myerson (1979)
- Examples of games of linking: Aumann and Myerson (1989) and Myerson (1991).
- Random/statistical linking due to Price (1972) and Erdos and Renyi (1960’s).

- A very active field of study since mid 1990’s; surveyed in recent books, Goyal 2007; Jackson 2008.

3. Unilateral linking

- Examples: Hyper links between pages, gifts, citations, peer to peer networks, phone calls…

- Unilateral linking is methodologically very convenient; it permits a thorough study of key questions:

  -- what is an equilibrium network,

  -- are equilibrium networks unequal

  -- are they socially efficient

  -- what are the dynamics of network formation.

3. Application: communication networks

- **Players:** Large number, N = 1, 2, 3,…,n)***.***
- **Strategy**: s_i defines a link with any subset of others.
- **Payoffs:** A link is costly; link between 1 and 2 gives 1 access to information which 2 has on her own, and information which she accesses via her links.

  Payoff *increasing* in people accessed directly/indirectly
  decreasing in the costs of links formed.

  Example: Payoff to player
  = [#players accessed] V – F. [# links formed]

3. Application: communication networks

- How do we solve this game?

- ***Nash equilibrium:*** A profile of linking strategies (s_1, s_2,,…s_n) one for each player with the following property: *every player is doing as well as possible, given what others are doing.*

- What is a Nash equilibrium of the game of network formation?

3. Application: communication networks

Some simple intuitions in example:

1*.* Suppose F > (n-1) V: then no linking: *Empty network.*

2. Suppose F < V: then player willing to pay to access everyone: *Connected network.*

3. Suppose V<F<(n-1)V: linking depends on other's behaviour:

If no one links, optimal to form no links: Empty network
If people form links then may be optimal to link.

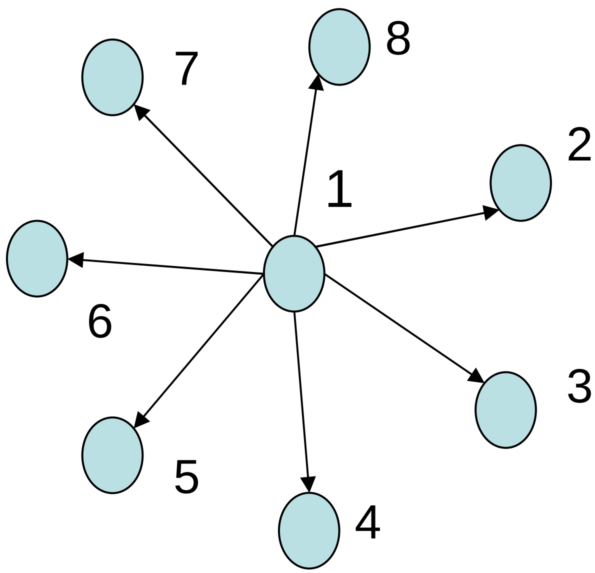
3. Application: communication networks

- **Theorem 1:** *Star is unique equilibrium architecture if value is falling in distance and linking is costly. [Bala & Goyal, 2000. Hojman & Szeidl '08, Ferri, '07].*

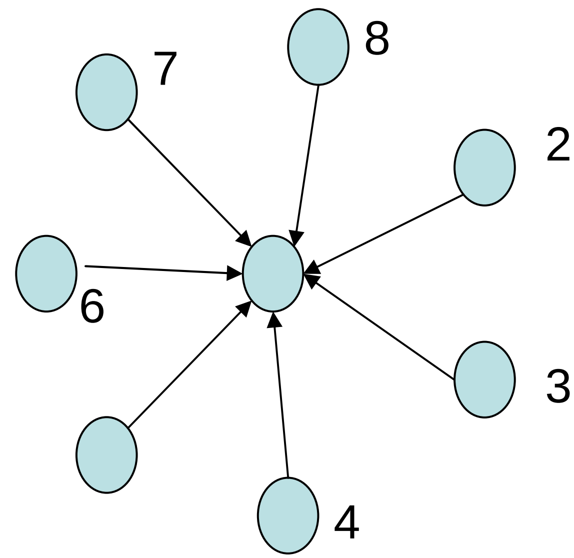
- Key intuitions:

  1. S*tar is equilibrium*: the spokes are accessing everyone with just one link, and everyone is close by.

  2. *Why is nothing else equilibrium*? Take any two end-players in a `tree' network. They have an incentive to get closer to the centre. Networks exhibit small world property.



Centre sponsored star



Periphery sponsored star

Equilibrium networks

3. Application: communication networks Dynamics

- Suppose players can observe the network and revise links over time. Network evolves over time.

Question: starting from an arbitrary network, will the dynamics converge, and what is the long run network?

Theorem 2:*Starting from any network, dynamics converge to the star network* [Bala and Goyal, 2000; Ferri, 2006].

General message: unequal degrees & short av. distances are robust features of incentive compatible networks.

3. Application: communication networks
Networking advantages

Does network degree and location confer advantages?

- Network formation leads to star in which the central hub player has privileged access to information. In general, the spokes pay for the links, and so hub gains both ways.

General message: *strategic networking can create large inequalities across players who are ex-ante identical.*

3. Application: communication networks Social efficiency

- Key idea: links are motivated by individual incentives. Linking generates externalities and spillovers on others. So there is a tension between equilibrium and socially desirable networks.

- In communication game: individual linking creates *positive* spillovers for others, and so individuals typically form less links THAN is socially desirable.

General message: *Equilibrium networks are under-connected, relative to socially efficient.*

4. Pair-wise linking

- A link requires the agreement of both parties. E.g., friendship, co-authorships, trade agreements, research alliances, buyer seller relations.
- Need for new solution concepts involving both non-cooperative and cooperative elements of game theory.
- Several developments in the theory and many applications…

4. Pair-wise links

- Basic idea: individuals propose links with others.
- A link between I and 2 is created if BOTH of them want to link.
- Myerson (1991) *link announcement game*.
- Solution concept: Nash equilibrium too weak as linking involves coordination between players.
- Supplement Nash Equilibrium with cooperative ideas

4. Application: research alliances

Leading firms in hi-tech industries rely on a combination of in-house and collaborative research. Biotechnology and pharmaceuticals; IT alliances. Hagedoorn (2004) shows

1. Firms in non-exclusive & extensive network of relations.
2. Research alliances have grown over time
3. Especially prominent in high technology sectors.
4. Core-periphery network architecture.

- Economic ideas: Strategic alliance among competitors.

  ---- alliance improves competitive position of partners

  ---- alters incentives of other firms to form alliances.

4. Application: research alliances

Firms bilaterally choose research links.

- Partners share technological information which lower costs of production. More links lead to lower costs, which leads to larger market share.
- However each link involves a fixed cost C.

Key features: 1. link decided bilaterally.

2. alliances arise in response to market pressures and they in turn define competition in networks.

4. Application: research alliances

Game of Network Formation:

- *Players*: N=(1,2,3,…n) firms
- *Strategies*: Each firm announces intention to form 0-1 links with others. A link is formed if both firms want it.
- *Payoffs*: A link costs F to each firm and lower their costs of production by c. Links formed define a network, which defines a vector of firm costs.

The gains from a link depend on market competition.

Strong competition: unique lowest cost firm makes profits
Moderate competition: lower costs imply higher profits.

4. Application: research alliances

- Solution of games with bilateral link formation?
- Nash equilibrium is too permissive: a coordination problem in bilateral link formation, firm 1 offers no link since it expects no one else to offer any!
- Way around: refine Nash equilibrium, require that no two unlinked players should have an incentive to form a link.
- Concepts: ***pair-wise stability, pair-wise equilibrium.***

4. Key concepts: Pair-wise stable and pair-wise equilibrium networks

- Network is *pair-wise stable* (Jackson & Wolinsky,1996) if
  1. no firm wishes to delete a link
  2. no pair of unlinked firms wishes to form a link.

- A network is *pairwise equilibrirum* if
  1. It constitutes a Nash equilibrium
  2. No pair of unlinked firms wishes to form a link.

Extensions to richer coordinated moves ---- extend to coalitional concepts. Jackson (2006, 2008) for surveys.

4. Application: research alliances

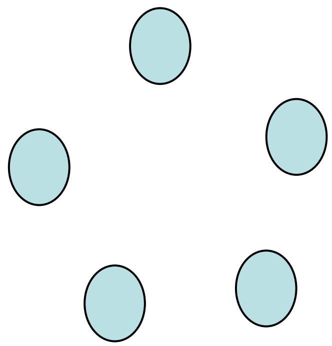
- **Theorem 3:** *Suppose F>0. With strong competition, empty network is unique pair-wise equilibrium. [Goyal and Joshi (2003)]*

  Intuition: in non-empty network, there is always a firm which forms link but makes no profits. Better to delete all links!

- **Theorem 4:** *Suppose F>0 and small. With moderate competition, complete network is unique pair-wise equilibrium. [Goyal and Joshi (2003)]*

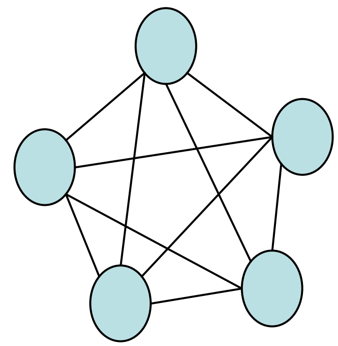
  Intuition: if two firms form links, gain at expense of other firms. Always form links.

General Message: Two-way influence… *markets shape networks and networks define market performance.*



Empty network

Strong competition



Complete network

Moderate competition

Pair-wise equilibrium networks

4. Application: research alliances Efficiency

Social welfare is sum of firm profits and consumers surplus.

Theorem 5: *With strong competition and small F, inter-linked star with two hubs is efficient*.

Theorem 6: *With moderate competition and small F, complete network is efficient.*

General message*:* *Moderate competition may attain greater efficiency… due to network effects.*

4. Application: research alliances

- We now turn to the case of high costs of forming links.

  Key issue: additional link creates a cost of F: how rewards from links are affected by # own & #others links:

  -- Whether marginal returns are increasing/decreasing in own links?

  -- Whether linking by others increases or decreases my returns?

  Key property: marginal payoffs are increasing in own links & decreasing in links of other firms.

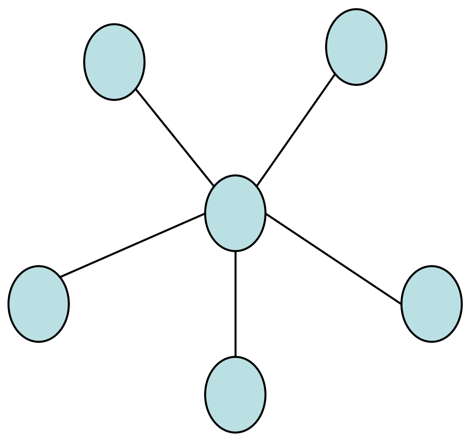
4. Application: research alliances Transfers, stars and market power

Theorem 7: *Suppose F>0 and firms subsidize other firms in links. Star and multiple hub networks are pair-wise equilibrium.*[Goyal and Joshi, 2003]

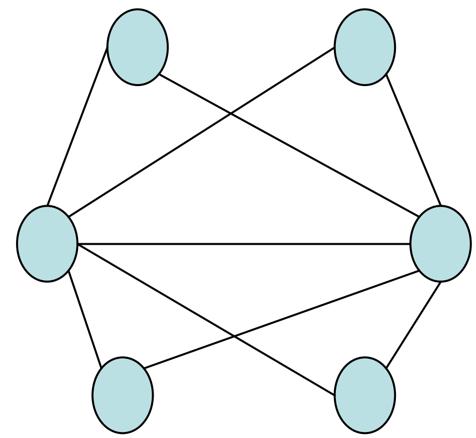
Intuition: Marginal gains from links are increasing in # of own links, so the central firm in a star has high marginal returns from new link. Many connections lower the returns to peripheral firms. So central firm subsidizes links with peripheral firms. Plus no other links!

Remark: The central firm with more links earns larger profits THAN peripheral firms. [Goyal and Joshi, 2003]

Key idea: Subsidizing links to create market power



A. Star Network



B. Inter-linked star (2 centres)

Equilibrium networks with transfers

4. Application: research alliances public policy

- Key externality: link between 1 and 2 lower profit of other firms; links create negative externality. *Firms create too many links.* [Goyal and Joshi 2003. Yi, 1998]

- Policy: Governments all over the world try and facilitate inter-firm collaborations; even subsidize them.

General message: our analysis suggests link taxing!

5. Quick summary

- The theory of network formation is concerned with understanding how networks arise out of strategic choices of players concerning link formation.

- The theory generates surprisingly sharp predictions on equilibrium networks: unequal degrees, small average distance, arise naturally. Good match with empirics.

- Strategic networking has powerful effects on payoff inequality as well as aggregate social welfare.

- Suggests role of policy – taxes and subsidies – to reorient network formation. [Taken up by the theory of mechanism design.]

6. Related themes
A. Weighted links

Existing work focuses on binary link setting: while most applications involve strength or depth of link…

E.g., bandwidth, time allocated to different social relations, strength of weak ties hypothesis (Granovetter, 1973).

- Formation of networks of weighted links is an important but very poorly understood process.
- References: Bloch & Dutta, 2008; Goyal 2005; Goyal, Konovalov & Moraga (2008), Bruckner 2005.

6. Related themes

B. Mechanism design

Consider a network formation game with players, linking strategies and payoffs.

Define a network as efficient if it maximizes aggregate payoffs.

Question: Is there an allocation function which respects plausible criteria – such as component wise budget balance, fairness etc -- and implements efficient networks?

Negative results due to Jackson and Wolinsky (1996); Dutta and Muttuswami (2001), significant follow up work e.g., Bloch & Jackson 2006.

6. Related themes
C. Choosing partners and playing games

Very large literature on local interaction and games in economics… survey by Young (1998), Goyal (2005).

In many settings, individuals choose partners & behavior.

Richer games: players choose links AND an action.

Networking and coordination: dramatic effects of costs of linking on coordination outcomes, e,g, Jackson & Watts 2002; Goyal & Vega-Redondo, 2006.

Networking and cooperation: beautiful results on networks and cooperation, Fosco and Mengel, 2009; Vega-Redondo 2006.

6. Related themes
D. Economic applications

- Market exchange: links between buyers and sellers, Kranton and Minehart (2001, 2003).

- Trade agreements: countries enter into free trade agreements to lower tariffs and facilitate trade. Goyal and Joshi, 2006; Furusawa and Konishi 2008, Zissimos, 2008.

- Non-entry agreements: firms collude to not enter each other’s markets. E.g. Belleflamme and Bloch, 2004.

- Financial networks: bank links to share risk; Babus 2008.

6. Related themes

D. Economic applications

- Co-authorships: linking with others means less time for existing projects but more projects…Jackson and Wolinsky 1996; Goyal, Moraga and van der Leij 2006.

- Internet backbone investments: will ISP’S have the right incentives to invest in backbones and links. Cremer et al (2000), Ignazio (2008).

6. Related themes

E. Strategic network design

General problem: Set of players N=(1,2,…n) face a set of nodes K=(1,2…k).

Application 1: Players choose links between the nodes and compete for traffic, e.g., airline networks.

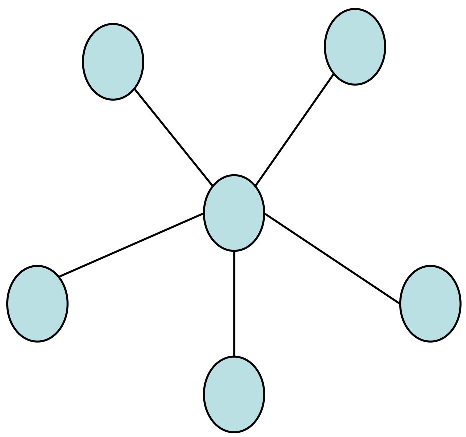
Application 2: one player chooses links to improve functionality, while second player seeks to lower performance. E.g., police and criminal networks, hackers and computer networks.

6. Related themes

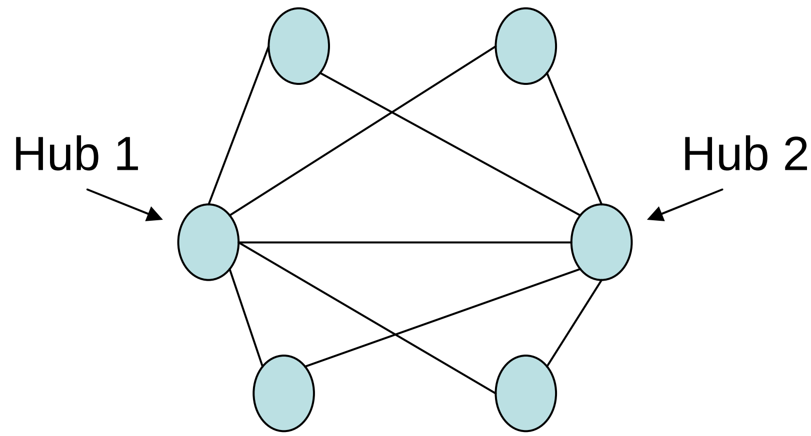
E. Strategic network design

- Airline networks… beautiful results obtained by Hendricks, Piccione and Tan (1996, 1997, 1999).

- **Monopoly problem**: single player chooses routes to operate between k cities. Hub-spoke network is optimal due to economics of traffic.
- **Duopoly problems**: with aggressive competition, single active hub-spoke networks, with moderate competition, multiple hub-spoke networks active.
- **Entry deterence**: a dominant carrier uses hub-spoke network to keep out entrants in the regional local routes.



Monopoly



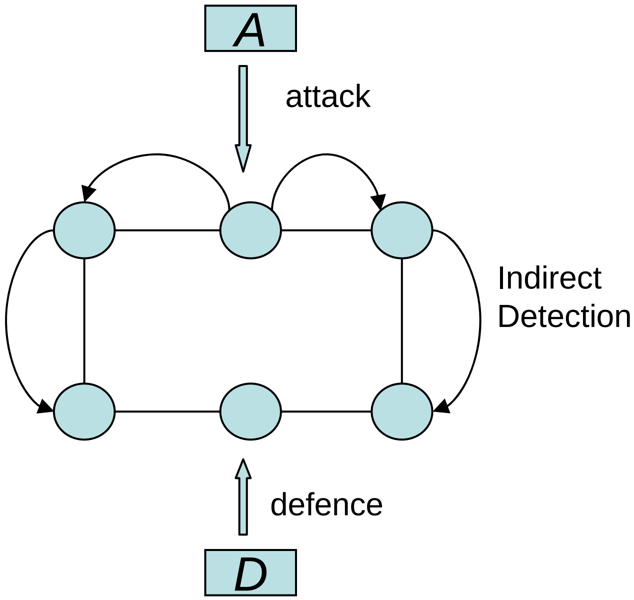
Duopoly with moderate competition

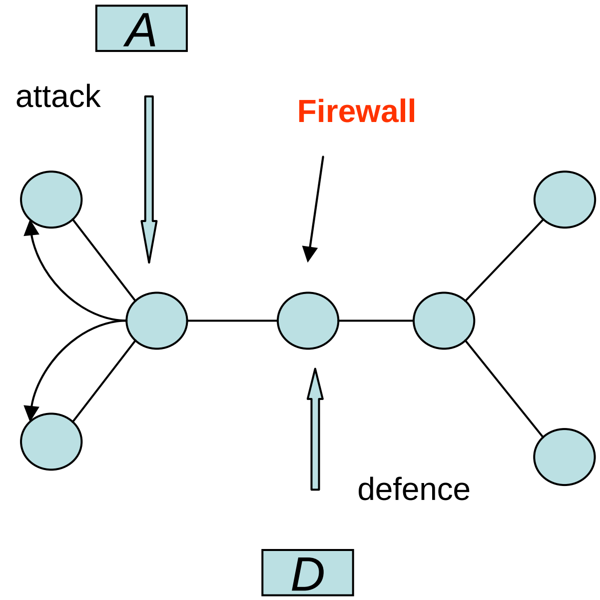
Equilibrium airline networks

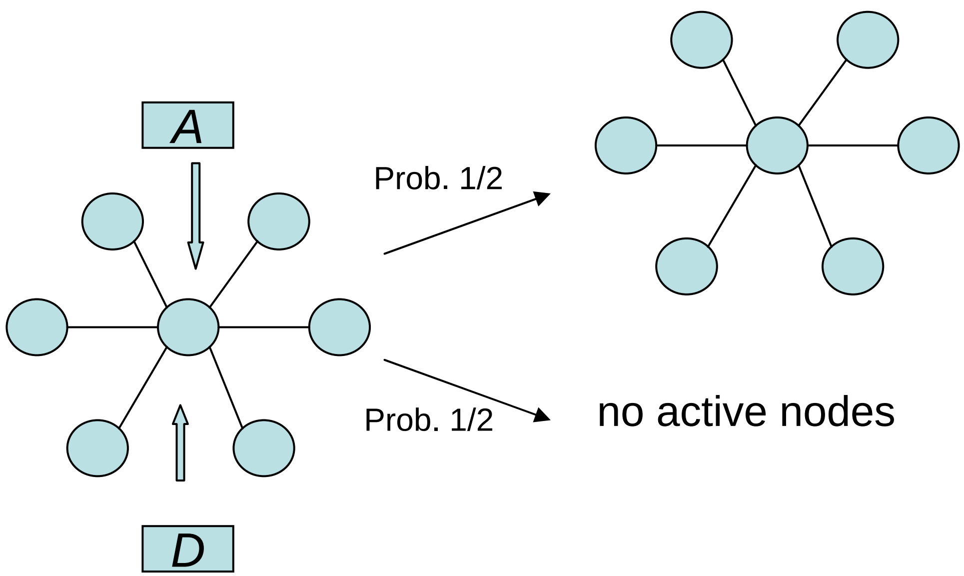
6. Related themes

E. Strategic network design

- Robust networks: designing networks faced with intelligent adversaries.
- Key idea 1: connections improve functionality but also make nodes more vulnerable to indirect infection.
- Key idea 2: suppose designer can protect a few nodes: protected nodes serve as firewalls, and block infection spread.
- Star network is robust in the face of intelligent attack and limited defence budgets.
- Baccara & Bar-Isaac 2008; Goyal & Viger 2008.


A
attack
Indirect
Detection
defence
D


A
attack
Firewall
defence
D


A
Prob. 1/2
Prob. 1/2
no active nodes
D

7. General theory: open problems

A. Dynamic network formation: network advantages suggest the pressure to pre-empt others in the creation of links. Very important open problem!

B. Network formation with large number of players: key role of incomplete information about players and about networks.

C. Networks and markets: traditionally economists focused on markets and ignored social structures. Recent work focuses on networks and ignores markets.

Urgent need to integrate networks and markets.