

Rational Expectations in Games

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American Economic Review, March 08

<http://www.ma.huji.ac.il/raumann/pdf/86.pdf>

Istanbul, 13 May 09

Heretofore, **Strategic Game Theory** has analyzed

- Games
via
- Equilibria

In contrast, we analyze

- Game Situations
via
- Individual optimizations
- Kadane & Larkey, 1982
ignored the **interactive nature of games**
- We **do** take it into account

Definition:

Game Situation :=

Game with belief hierarchies

Assumptions:

1. Common Knowledge of Rationality (CKR)
2. Common Priors (CP)

Definition:

A *rational expectation* of a player in a game G is her expectation in some game situation based on G , with CKR and CP.

Theorem A:

Every rational expectation in a two-person zero-sum game is that game's value.

Traditional arguments for the value:

1. Guaranteed expected payoff
2. Equilibrium

Theorem B:

The rational expectations of a player in a game are precisely her conditional payoffs to correlated equilibria in the “doubled” game: that in which each of her strategies is written twice.

	L	R
T	6,6	2,7
B	7,2	0,0

	L	R
T	$\frac{1}{2}$	$\frac{1}{2}$
B	$\frac{7}{8}$	$\frac{1}{8}$

	L	R
T	$\frac{1}{2}$	$\frac{7}{8}$
B	$\frac{1}{2}$	$\frac{1}{8}$

	L	R
T	$\frac{7}{22}$	$\frac{7}{22}$
B	$\frac{7}{22}$	$\frac{1}{22}$

	L	M	R
T	0,0	4,5	5,4
C	5,4	0,0	4,5
B	4,5	5,4	0,0

	L	M	R
T1	0	0	1
T2	0	$\frac{2}{3}$	$\frac{1}{3}$
C	$\frac{1}{2}$	0	$\frac{1}{2}$
B	$\frac{1}{2}$	$\frac{1}{2}$	0

	L	M	R
T1	0	0	$\frac{1}{4}$
T2	0	$\frac{1}{2}$	$\frac{1}{4}$
C	$\frac{1}{2}$	0	$\frac{1}{2}$
B	$\frac{1}{2}$	$\frac{1}{2}$	0

	L	M	R
T1	$\frac{1}{12}$	0	0
T2	$\frac{1}{12}$	$\frac{1}{6}$	0
C	$\frac{1}{6}$	0	$\frac{1}{6}$
B	0	$\frac{1}{6}$	$\frac{1}{6}$

In Economics, “a rational expectation is one that is the same as the prediction of the relevant economic theory” (Muth, 1961).

Slightly rephrased: the players know the relevant theory (and of course, that it applies to the situation at hand).

In games, the relevant theory takes all players to be rational.

So all players know that all are rational.

So all know that

So all know that

...

So, CKR.

Next, the “relevant” theory may be thought of as yielding a probability distribution p on profiles of beliefs of the players.

But each player knows her own beliefs.

So her beliefs are the conditional of p given her knowledge.

That is CP.